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Mathematics News Letter

SUBSCRIPTION $1.00 PER YEAR Single Copies . . 15c

Published eight times per year at Baton Rouge, Louisiana.

Dedicated to mathematics in general and to the following aims in particular: (1) a study of the common problems of secondary and collegiate mathematics teaching, (2) a true valuation of the disciplines of mathematics, (3) the publication of high class expository papers on mathematics, (4) the development of greater public interest in mathematics by the publication of authoritative papers treating its cultural, humanistic and historical phases.

VOL. 8 BATON ROUGE, LA., JANUARY, 1934 No. 4

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The Universal March of Mathematics

Wherever human struggle is, THERE *must human reason function, however little or however much, however feebly, or however well. Though degree of logical precision demanded may vary in the problems that loom, basic processes in their proper solution must always be the same. Logical procedure in Euclid or in the calculus must forever have its counterpart in all other problem domains, however discrepant or even contrasted may be the problem materials.*

Thus always is the march of mathematics from (a) hypotheses to (b) objective, by way of (c) analysis of hypotheses, (d) analysis of objective, (e) judgment of proper working principles, following (f) a conceived theory of connection between the data and the objective.

Basically, these processes are no more present in the solving of the arithmetic, the geometric, or the electric problem, than they can be made to be visible in the astronomical, the biological, the industrial, the game problem or the language deciphering problem.

There is a fascination in observing their march in Sir Isaac Newton's solution of the most stupendous problem that ever challenged the human intellect, the problem: To show the universality of gravitation.

NEWTON AND THE PROBLEM OF GRAVITY.—*He started from the following established data:*

(a) *Gravity holds in the neighborhood of the earth.*

(b) *A stone dropped from a point in the neighborhood of the earth falls through 16 feet in the first second.*

(c) *The path of the moon about the earth is approximately circular.*

(d) *The distance from the earth's center to the moon is known.*

(e) *The earth's radius is known.*

(f) *The time of the moon's rotation about the earth is known.*

(g) *If a straight line could be drawn from the sun to any one of the planets, the motion of the planet is such that the straight line would sweep over equal areas in equal times.*

(h) *The orbit of each planet is an ellipse, with the sun at one of its foci.*

(*i*) *The squares of the periods of the planets are to each other as the cubes of the major semi-axes of their respective orbits.*

His WORKING PRINCIPLES *were: The principles of mathematics.*

His OBJECTIVE *was:*

A demonstration that the same law that holds the planet in its orbit also determines the falling of the stone towards the earth and the rotation of the moon about the earth.

From data (*g*), (*h*), (*i*), *known as Kepler's Laws, he, with other mathematicians, had already inferred that the sun attracted each of the planets with a force inversely proportional to the square of its distance. With this knowledge, he* CONCEIVED *the idea of an arithmetical proportion, made up of the following elements: the distances given in data* (*d*) *and* (*e*)*; the measure of the earth's attraction for a stone near its surface, namely, a pull of 16 feet in the first second* (*data* (*a*) *and* (*b*))*; a measure, x, of the hitherto undetermined force constantly deflecting the moon from a straight path tangent to its circular orbit* (*datum* (*c*)). *Data* (*a*) *and* (*c*), *with elementary mathematics, being sufficient to compute the value of x, if the force represented by x were truly the result of the earth's pull on the moon, and if the law of such attraction should indeed be identical with the law of attraction of the sun for a planet, then, x should be found to satisfy the relation* $x : 16 :: r^2 : d^2$, *in which r is the length of the earth's radius, d the distance of the moon from the earth's center.*

The result of Newton's first test of the correctness of his conceived proportion was disappointing. His computed value for x did not satisfy the proportion. The failure was due to an erroneous value for r. Thirteen years later, certain geodetical researches of Picard disclosed a corrected value of the earth's radius. With this new number placed for r in the proportion $x : 16 :: r^2 : d^2$, *that proportion was satisfied, and his objective was established.*

A ZOOLOGICAL PROBLEM.—*Many years ago California faced a problem. An insect pest, known as the fluted scale, was destroying the orchards of the state. Professor C. V. Riley, Government entomologist, was appointed to devise means for getting rid of the pest. How was the problem to be solved? By sedulous use of observation and induction he got possession of the life-history of the insect, all the facts of which were carefully recorded. The data thus obtained were used. Sprays*

were formed, thoroughly adapted to destroy great numbers of the insect without injuring the trees, and without entailing much expense. But the fruit-growers would not use the spray because they lacked confidence in its value. Professor Riley had tested its efficacy, but the HUMAN *link was needed to connect the scientific formula with its* APPLICATION, *and, because that link depended, not on necessity, but on uncontrolled volition, the solution of the problem by the spray route* FAILED.

Then followed wider induction, resulting in fresh facts from which new trains of implication were started. The pest was found to be as bad in South Africa and in New Zealand as it was in California. The native habitat of the scale was Australia. But, in Australia, while it existed, it did not exist as a pest. From this last association of facts Professor Riley began a new DEDUCTIVE *trail. He conceived a theory as follows: In all the countries in which the scale is found, it exists as a pest except in Australia. If, then, the cause that hinders it from being a pest in Australia could be found, it might be possible to apply the same cause to stop the pest in California.*

To test out his conceived theory he sent men to Australia to investigate. They found in Australia a beetle that lived solely on the fluted scale, and they found that it had a marvelous power of multiplying itself. A vast number of specimens of this beetle were placed on ice in little tin boxes, sent to California, and liberated in the midst of the orchards. Within a year's time the voracious appetite of the beetle for the fluted scale, together with its enormous power to multiply, had resulted in the complete elimination of the pest from California's orchards. A train of NECESSARY *implications had been found, the last of which was the objective Professor Riley had been seeking.*

A LANGUAGE PROBLEM.— *Data: A Latin sentence with adequate context.*

Objective: To find its English translation.

Working Principles: Those embraced under the grammar and idiom of the Latin language, with a Latin-English dictionary.

Connecting With the Objective: Using observation, comparison, analysis, induction, deduction, experiment, to select from grammar and idiom and dictionary those implications of meaning correspondent to the given Latin words, and yielding an English translation that is definite and consistent in its meaning taken with context.

A CHESS PROBLEM.—*Data: A chess board, to be used as in a game of chess; black king, black queen; white king, white queen, white bishop, white knight; all occupying certain squares on the board.*

Working Principles: The usual prescribed modes of moving the king, the queen, the bishop, the knight; the usual meaning of "mate," or check-mate.

Objective: A check-mating of the black king by the white pieces, number of moves limited to THREE.

Connecting With the Objective: Using observation, comparison, analysis, induction, deduction, experiment, to find a succession of three moves, and three only, that will discover a "mate" by white.

—S. T. S.

The Jackson Meeting

The Louisiana-Mississippi section of the Mathematical Association of America will hold its annual meeting jointly with the National Council of Teachers of Mathematics and the Mississippi Academy of Science at Jackson, Miss., on March 23 and 24.

We shall have the unusual opportunity this year of having as our guest speaker Dr. Arnold Dresden, President of the Mathematical Association of America. During his Easter vacation Dr. Dresden will make a combination trip into the South. He will speak before the Louisiana-Mississippi Section on March 23 and 24 and before the Southeastern Section on March 30 and 31 at Birmingham.

A committee composed of Doctor McCoy, Doctor Mitchell, and Mr. Beckett will make a great effort to give us a fine local atmosphere at Jackson. We had a splendid meeting in Jackson in 1928.

Again, friends and members of the Association, let us have at Jackson two days of friendly intercourse and intellectual fellowship. Let us equal the record of that latest meeting at Jackson. Indeed! Let us surpass that excellent meeting in enthusiasm and in attendance.

P. K. SMITH, *Chairman.*

Goals in the Solution of Geometric Problems

By Sophie Wright High School
New Orleans, La.

Quite often I find myself telling pupils that their troubles are few if once they find out how to study—how to study Geometry. By this I mean that the problem of teaching Geometry is essentially one of teaching pupils how to study—how to attack their problems in order to arrive at a satisfactory conclusion.

It may be pointed out to the student that, in order to attain a reasonably broad conception of study, one needs to look away from the school to the methods followed by efficient workers in the various walks of life. Here is found a difficulty to be overcome, a particular need to be satisfied, or a problem to be solved. In other words, there is the presence of a specific purpose or motive in overcoming the difficulty, satisfying the need, or solving the problem. Before any real work can be done in the solution of an original in Geometry, the student must have a more or less specific aim, and must see that there is a real problem before him. He must get clearly in mind the purpose of the problem he has to solve. This understanding of the purpose or aim is quite a necessary factor in disposing a pupil to respond whole-heartedly to any instruction that may be given him.

This purpose or aim should be set up quite clearly in the lesson assignment. Here is the teacher's opportunity to prepare his class for constructive, independent study. The student may be led to find the purpose or aim himself by being required to read carefully and intelligently each statement of his problem. This has not been done thoroughly and properly until he is able to state exactly what is given as a basis for his thought and concisely the purpose of the particular problem in hand. The teacher who can lead the student to do this for himself is doing some real constructive work even though such a process may be a tedious one and somewhat long-drawn out. The patience of most teachers is entirely too inelastic to withstand this strain. Instead of holding out and leading the student to do this thing for himself, we grow weary and do this part of the work for those whom we should lead to do things for themselves.

Once the student has accomplished this first step, he has filled himself with a real desire to work through his own interest and attention

in stating for himself the thing to be accomplished in a manner which moves him to make further effort in the solution of his problem. This next step involves the procedure to be followed in reaching the necessary conclusion. But, before this can be done, all available information must be assembled. Here, again, the student should be taught to do for himself. At this point, skill in making correct application of conclusions previously drawn—the using of facts at one's finger tips—is essential. Some pertinent facts which relate to the problem in hand may be suggested by well directed questions on the part of the skillful teacher, but the pupil should be held responsible for other important facts to be gathered. Of course, many ideas will intrude which have no direct bearing on a particular problem. But the teacher must bend every effort to have the student connect his material definitely and directly with the problem at hand. He may set up a series of questions such as: Do such and such facts relate to the problem? Do they aid in clearing up any of the steps of the solution? Do they seem to contradict any other ideas or facts which have been assembled? All such questions as these will help the pupil to set a standard of judgment in regard to relevant facts and eventually lead him to formulate these and other pertinent questions for himself. If, thus, the student can be led to judge the worth of facts, to discard those which are irrelevant or unimportant, and to establish a logical correlation between the facts themselves, between the facts gathered and the purpose of his task, as well as between those facts and the procedure to be followed, he has travelled far in the solution of his problem.

Geometric ideas, more or less, through a natural process become associated in groups but, in purposive thinking such as is required in the solution of originals in Geometry, this process must be consciously aided. Some one has said that it is the task of the teacher to train the child to analyze data, to pick out important points, to eliminate irrelevant material, and finally to arrange the whole according to some pattern that will bring out the real purpose and lead to the solution of some definite concrete problem. In no phase of the teaching process is this truer than in the solution of Geometric problems. Here the student must be able to use diagrams and figures intelligently and to see that the search for data has gone far enough. Summaries or generalizations which involve intelligent organization weave the content of our subject matter into a fabric which it is easy for the pupil to hold and to utilize when need arises.

With all available material at hand, the student is now ready for his analysis in order to prepare the way for generalization. But this process of analysis and synthesis should be made systematically; otherwise, the immature pupil's mind may rush into a conclusion which represents his general impression rather than a proper generalization of his Geometric experience. The child must be encouraged to summon his fullest resources, bringing all his related knowledge to bear upon each particular situation before attempting to generalize. In other words, judgment should be suspended until sufficient data have been accumulated to justify a correct and intelligent conclusion. The inability of the average third year student in high school to do this, together with his spontaneous uncurbed outbursts, often lead him to give answers that have no very thorough basis of evidence; he often builds elaborate inferences on the flimsiest of foundations. The capable teacher can often reduce this tendency to a minimum by insisting that every statement made in the process of proof be supported by good and sufficient reason; in any event, the student must be led to assure himself that he is right. While, from having his work approved by others to being able to check it himself, is a long, long journey, this is the one step, above all others, that instills confidence and leads to independent work on the part of the pupil.

We are frequently told in certain quarters that more and more the school should be a place where pupils study and experiment and debate rather than a place where they go to recite lessons. In no phase of our school life is this principle more applicable than in teaching the student to attack and arrive at a satisfactory solution of Geometric problems. In such a scheme, whether it be accomplished by proper assignment and home study, or through really efficient supervised study, the teacher must be a real, live, consulting expert in assisting the student to attack his problems. This, to be sure, does not mean an injudicious helping of students, which, too often, develops the mirror-minded pupil. But, on the contrary, it requires that the teacher must stimulate the pupil to go through the necessary process of learning his lessons for himself. It should be the function of the teacher to make the pupil independent of the school and its ministrations as fully and as early as possible. In the solution of each and every problem the teacher's ambition must be to equip his pupils with definite aspirations for knowledge and with the ability to secure it for themselves. When the high school receives the child he does not know how to approach a new assignment; he has difficulty in picking out the central idea or

in deciding what is of major or minor importance. He knows little of how to gather information, nor does he know how to organize what he has or formulate it in verbal or written language. In short, we are told, he does not know how to think.

If, through the solution of original exercises, the teacher of Geometry is able to do some of these things, he is in a measure meeting the challenge of the modern school to teach the pupil to do all of these things for himself. If the child learns how to direct his observations, to read his books intelligently, to organize his facts, and to apply knowledge, he has gained power to meet his needs as they confront him.

In short, if the child can be taught how to study, the solution of his problems, in Geometry or elsewhere, will very largely take care of themselves.

The Unit Assignment in Algebra and Geometry

By William A. Payne
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This subject can hardly be developed without a consideration of the following problems:

I. What is meant by the unit assignment?

II. What is the value of the unit assignment as compared with the daily recitation plan?

III. Do algebra and geometry lend themselves to this form of procedure?

After a necessarily brief discussion of these topics, I shall quote a specimen assignment in algebra from a National Survey and also give my plan for an assignment in geometry.

The learning unit* as defined by William C. Ruediger is "any division of subject matter, large or small, that when mastered gives an insight into, an appreciation of, or a mastery over, some aspect of life." This term then would cover the unit assignment or any one of its sub-

*Ruediger, William C.—The Learning Unit School Review 40, pp. 176-81, March '32.

divisions. In this discussion, however, the "unit" will be used to designate one of the subdivisions of the assignment rather than the whole.

The unit assignment, as applied to mathematics, is the teacher's plan for the pupils' activities and experiences which in his judgment are best fitted to produce the mastery of particular related units grouped under one general topic. It is usually in the form of a guide sheet, and consists of: (1) directions for study; (2) references; (3) a list of supplementary projects; (4) an outline of essentials (sometimes the maximum and sometimes the minimum); and (5) a tentative time allotment. There is often added in addition to the above, an introductory paragraph to stimulate interest and curiosity; necessary explanations; helps in the form of enumeration of special difficulties; additional optional work; and a test on the assignment. Some also add reteaching and retesting. The methods used in covering the assignment are left to the individual teacher.

A National Survey of Education† conducted in 1928 by the United States Department of the Interior under the direction of Dr. Leonard V. Koos, of the University of Chicago and at the suggestion of the North Central Association of Colleges and Secondary Schools, found that the unit assignment is being widely used all over the country, particularly in the larger schools and those of the reorganized type; and in practice, differentiated assignments, whether listed under the Project, Problem, Contract, Laboratory, Morrison, Dalton or Winnetka plans were all characterized by the unit assignment.

The daily recitation plan emphasizes the teacher rather than pupil participation and dates back to the days when the usual lesson assignment was, "Take the next ten pages," or "Work from the first through the tenth examples on page 67." Such assignments did not stimulate interest and curiosity, which form the basis for intelligent and purposeful study. They were sufficient, perhaps, a generation ago when the high school pupils were a more or less highly selected group capable of making their own way, but with the heterogeneous grouping of today the assignments must provide for individual differences, guide, direct, and inspire. The unit assignment does all these, because by setting a definite problem it fosters pupil activity and at the same time is elastic enough for even the slow pupils to cover at least part of the minimum essentials, which encourages further effort on their part. Therefore it is psychologically sound.

†Bulletin 1932, No. 17; Monograph No. L3, p. 355, U. S. Department of the Interior.

Does this mean that all of our subject matter should be handled through the unit assignment? No, some units may best be handled by the daily recitation plan; such as, the axioms and definitions in Geometry, for instance. This calls for judgment on the part of the individual teacher.

An interesting experiment‡, to determine the comparative values of these two plans, was conducted in Queen Anne High School in Seattle, Washington, in 1928-29, during the fall semester. Four classes of unselected groups in General Science were used. They had the same teacher, used the same texts, same equipment, and in so far as was possible all the factors, except the method, were kept identical. Two groups followed the unit assignment, and two the daily recitation plan. Results were tabulated and compared according to matched partners corresponding in age, sex, I. Q., term grades and semesters in school. The study showed a slight general advantage in favor of the unit assignment plan, as seventeen made more progress than their matched partners in the daily recitation group. Neither yielded better results consistently, but varied with the type of subject matter. Factual knowledge was best taught by the unit assignment. Hence, the authority for my above statement that some ideas can best be handled by the old time method, as all do not lend themselves readily to the unit assignment.

The subject matter of mathematics, due to its logical sequence, lends itself very readily to the unit assignment. In fact, the text books in both algebra and geometry are in themselves guide sheets, as related units are already grouped under topic headings and the teacher can very easily round out the assignment. In the 362 schools studied in the above mentioned National Survey, about eighty percent of their several offerings were presented by the unit assignment, and mathematics ranked fourth highest in the subject matter fields, averaging fifty-seven percent; plane geometry, seventy-three; algebra, sixty-three; solid geometry, fifty-five; and trigonometry only thirty-five percent. The assignments in these subjects, of course, are briefer than those in history and English.

I shall now quote a unit assignment in algebra which is adequate yet brief. It was submitted by the South Philadelphia High School

‡Shelton, A.—An Experimental Study of the Daily Recitation Versus the Unit Plan—Ed. Journal, Vol. 38, '30, pp. 694-699.

for girls in Philadelphia, Pa., and published in the report of the same survey*:

Algebra I. Time: one week. Text: Durell and Arnold, Shorling-Clark-Lidell; Instructional Tests in Algebra. Fractions (all work in Durell and Arnold).

1. Study p. 160. Note that fractions in Algebra in general have the same properties as fractions in Arithmetic. Transformation of fractions.
2. Reduction of fractions to lowest terms. In exercises 86 and 87, pp. 161-62, work the odd numbered problems.
3. Reduction of an improper fraction to a mixed number. Study p. 163 and work examples 2, 4, 5, 7, 10, 12, 14, 20 and 22 in exercise 88.
4. Reduction of mixed expressions to a fraction. Study p. 163 and work examples 6, 9, 11, 14, 19 and 20 in exercise 89.
5. Do you remember how to do addition, subtraction, multiplication and division of whole numbers? Test yourself by working example 14, p. 44; example 37, p. 48; example 30, p. 82; example 14, p. 82; example 20, p. 87. Work exercises 28 and 30, p. 19.
6. Maximum assignment; Study pp. 176 and 177, Part II. Work examples 1-3, p. 178; example 1-3, p. 180; example 16-18, p. 180.

As far as Geometry is concerned, I should like to submit my own plan:

Geometry I. Time: one week. Text. Wells and Hart, Modern Plane Geometry; Avery, Geometry Workbook. Congruency of triangles.

1. Study p. 25. Note: In proving triangles congruent, find first the equal sides. Study the figure to find any sides that may be equal by identity. If you can not find any equal sides, you cannot prove the triangles equal. Before your work is complete, you must be able to name the three parts of one triangle that are equal respectively to the three parts of the other.

*National Survey, Bulletin 1932, No. 17, Monograph No. 13, pp. 331-32; 355.

2. Triangles having two sides and the included angle equal. Study pp. 26-28 and work examples 7, 10, 13, 19 and 20.
3. Triangles having two angles and the included side equal. Study p. 30 and the lower half of p. 31. Draw figures for exercises 30 and 34. Work exercises 31, 33 and 37. Complete statements on pp. 23 and 26 of work book.
4. Triangles having three sides equal. Study p. 35. Draw figures for exercises 50 and 52 on p. 36. Work exercises 47 and 51. Complete the statements on pp. 43 and 44 of the work book.
5. Answer questions of exercises 56-60 on p. 36 of text. Write the theorems for the exercises on pp. 43 and 44 of the work book.
6. Optional work: exercises 7-9 on p. 272 of the text.
7. Method test on pp. 45 and 46 of work book. (Regular trimester test to be taken later).

Present day educators attach great importance to the pupil's attitude towards his task, usually referred to as "interest" or "whole-hearted, purposeful activity." Therefore it would appear highly desirable to know the pupils' reaction to the unit assignment. An attempt was made to discover the reaction of boys and girls of various levels of intelligence, accomplishment and application to this type of assignment. What do they consider the advantages or disadvantages of this plan of instruction as contrasted with the class room procedure of the daily recitation plan? The study† was conducted by Thomas W. Harvey of the Memorial High School, Planesville, Ohio. We are interested only in the results obtained by the inquiry form filled out by 75 pupils in three tenth-grade sections in plane Geometry. Forty-five distinct advantages of the unit assignment were listed against ten disadvantages. It was the general concensus of opinion that the unit plan places the pupil on his own responsibility, he may work at his own rate of speed, is the only one to blame if he does not get a good mark, and "makes failure unnecessary for even a slow pupil if he is willing to work."

Since the unit assignment provides for individual differences, holds the interest of the pupils, and produces better results than the traditional daily recitation plan, it should be used more extensively in both algebra and geometry.

†Billet, Roy O.—High School Pupils' Opinion of the Unit Plan, School Review, Vol. 40: pp. 17-32, Jan. '32.

Divisibility Rules by the Remainder Theorem

By Robert Gaskell
Ann Arbor, Michigan

Numerous methods have been used in the past for developing divisibility rules. Dickson, in his "History of the Theory of Numbers,"* gives a very good history of these methods. The remainder theorem affords a very neat and systematic method of developing a large number of the rules.

Given a number base N, any integer I can be written as a polynomial in N, that is:

(1) $$I = f(N) = a_0N^n + a_1N^{n-1} + \ldots\ldots\ldots + a_{n-1}N + a_n.$$

If we divide this polynomial by h, we will have, by the remainder theorem, a remainder

$$f(N-h) = f(k) = a_0k \; + a_1k^{n-1} + \ldots\ldots\ldots + a_{n-1}k + a_n,$$

where $N = h + k$. Now this is only an "algebraic" remainder, and if this process is applied strictly to a numerical example, the algebraic remainder found will be often actually larger than the divisor. It is easy to prove, however, that the "numerical" remainder of an algebraic remainder is the same as the numerical remainder of the given number.

The difference between algebraic and numerical remainders can best be shown by means of an example. If we are to divide 152 by 7, we get 5 as a numerioal remainder. $(152 = 21 \cdot 7 + 5)$. But if we use the method of the remainder theorem

$$I = f(10) = 10^2 + 5 \cdot 10 + 2$$

$$R = f(3) = 3^2 + 5 \cdot 3 + 2 = 26.$$

This number 26 is called the algebraic remainder. When it is divided by 7, it also gives a numerical remainder of 5. The algebraic remainder, then, will be just as useful to us as the numerical remainder.

*Vol. I, p. 337 et seq.

See also: Chrystal, "Algebra," Vol. 2, p. 532; Fitz-Patrick, "Exercises d'Arithmetique," p. 225 et seq.

II.

In applying the remainder theorem to the 10 base system, we first write down an expression for any number in the decimal system:

$$f(10) = a_0 10^n + a_1 10^{n-1} + \ldots\ldots\ldots + a_{n-1}10 + a_n.$$

To proceed with our tests we first take as a divisor the number 9. If we divide f(10) by (10−1) we have for the algebraic remainder

$$f(1) = a_0 + a_1 + a_2 + \ldots\ldots\ldots + a_{n-1} + a_n$$

This number has the same numerical remainder as the given number, and it represents the sum of the digits of the number. We may therefore state the rule for divisibility by 9: If the sum of the digits of any given number is divided by 9; it will leave the same remainder as the number itself when divided by 9.

For 3 as a divisor we obtain as a first algebraic remainder

$$f(7) = a_0 7^n + a_1 7^{n-1} + \ldots\ldots\ldots + a_{n-1}7 + a_n.$$

This is again an expression of type (1), and we can repeat the process and obtain as a second algebraic remainder

$$f(4) = a_0 4^n + a_1 4^{n-1} + \ldots\ldots\ldots + a_{n-1}4 + a_n;$$

and again dividing this number-expression by 3, we obtain finally:

$$f(1) = a_0 + a_1 + a_2 + \ldots\ldots\ldots + a_{n-1} + a_n$$

This leads us to the same rule for 3 as we derived for 9, the "sum rule."

The next number to be considered is 11. Its general algebraic remainder is:

$$f(-1) = \pm a_0 \mp a_1 + \ldots\ldots\ldots + a_{n-2} - a_{n-1} + a_n.$$

This gives us the "difference rule;" that is, if the difference between the sum of the digits occupying the even positions and the sum of those occuping the odd positions is divided by 11, it will leave the same remainder as the given number, when it is divided by 11.

The tests for some other numbers involve more complicated rules, which will now be developed. If we take 7 as a divisor, we have:

$$f(3) = a_0 3^n + a_1 3^{n-1} + \ldots\ldots\ldots + a_{n-1}3 + a_n.$$

The coefficients (or digits) will then be multiplied by factors which are powers of 3. Reversing the polynomial

$$f(3) = 3^0 a_n + 3^1 a_{n-1} + \,.\,.\,.\,.\,.\,.\,.\, + 3^{n-1} a_1 + 3^n a_0.$$

We can now find the actual values of $3^0, 3^1, \ldots 3^n$; and evaluate this expression term for term and then add. But we can simplify this process. It is easy to see that the remainders obtained by dividing the consecutive powers of 3 by 7 form a repeating sequence:

1 3 2 6 4 5 1 3 2 6 4 5 1 3 2 . . . .

If these numbers are substituted for their corresponding powers of 3, it will not change the numerical remainder. We then formulate the rule: Multiply the units digit by 1, the tens by 3, the hundreds by 2, etc. . . according to the sequence found, and add these products. If this sum is divided by 7, it will leave the same remainder as the given number, when it is divided by 7.

This rule is impractical when applied to numbers of any size; but it might be applied with some advantage to 3 digit numbers. We can find, by resorting to higher number bases which are powers of 10, rules which do not involve such a sequence, but which are still impractical. These rules will come later in the discussion when general rules to be presented will simplify the task.

A very simple sequence to be applied in the same manner can be found for 6. It is derived from the powers of 4, in the same way as the sequence for 7 was derived from powers of 3. The sequence is

1 4 4 4 4 . . . . .

This sequence makes a very good test for divisibility by 6. It has an advantage over the common method of testing with 2 and 3 separately, because by the sequence rule we find the remainder which would be obtained, and not just whether or not our number is divisible by 6. This same argument applies in the case of the sequence test for 12:

1 −2 4 −8 4 −8 4 . . .

Other sequence tests which can easily be verified are:

$$13: \quad \dot{1}\ -3\ 9\ -1\ 3\ -9\ \dot{1}\ \ldots\ldots$$

$$8: \quad 1\ 2\ 4\ \dot{0}\ \dot{0}\ \ldots\ldots$$

$$4: \quad 1\ 2\ \dot{0}\ \dot{0}\ \ldots\ldots$$

The rules for these divisors are now self-evident. These rules for 4 and 8 also show that the last three digits of a number determine divisibility by 8, and that the last two digits determine possibilities by 4.

It is interesting to notice the forms which these various rules take when we use a number system not based on the number 10. For instance, the number-base 7 gives us the following rules:

2,	sum or difference rule;	3,	sum rule;
4,	difference rule;	5,	no rule;
6,	sum rule;	8,	difference rule.

For the number 5 apparently no simple rule can be found by the remainder theorem. However a sequence rule may be found for this number . It is

$$\dot{1}\ 2\ 4\ 3\ \dot{1}\ 2\ 4\ 3\ \dot{1}\ \ldots$$

In the same way we can find rules for divisors in other number-base systems.

III.

These sum and difference rules can easily be generalized. The following rule can easily be proved:

If $N-1$ is divisible by the given divisor, that divisor will require the sum rule;

If $N+1$ is divisible by the given divisor, it will require the difference rule;

If neither $N+1$ nor $N-1$ is divisible by the given divisor, then neither the sum nor the difference rule will be applicable to that divisor.

PROOF

If $(N-1)$ is divisible by the given divisor we know that

$$N-1=q\ .\ d.$$

If we have:

$$f(N)=a_0N^n+a_1N^{n-1}+\ .\ .\ .\ .\ .\ +a_{n-1}N+a_n,$$

we may divide by d, and get

$$f(t)=a_0t^n+a_1t^{n-1}+\ .\ .\ .\ .\ .\ +a_{n-1}t+a_n;$$

where $t+d=N$. Or this may be written:

$$f(N-d)=a_0(N-d)^n+a_1(N-d)^{n-1}+\ .\ .\ .\ .\ .\ +a_{n-1}(N-d)+a_n.$$

Now let $N-d=N_1$; $N_1=t_1+d$. Then, dividing again by d:

$$f(N_1-d)=a_0(N_1-d)^n+a_1(N_1-d)^{n-1}+\ .\ .\ .\ .\ .\ a_{n-1}(N_1-d)+a_n.$$

But since $N-d=N_1$,

$$f(N_1-d)=f(N-2d).$$

If we let $(N_1-d)=N_2$, we can arrive at a third function, $f(N-3d)$, etc. . . . , finally obtaining $f(N-q.d)$as a remainder. But this last remainder is $f(1)$ and it reduces to:

$$a_0+a_1+a_2+\ .\ .\ .\ .\ .\ +a_{n-1}+a_n$$

Therefore the test for the divisor d can be applied to $f(1)$, and the sum rule can be used on the original number.

The proof of the second part of the rule is exactly analogous to this proof, the final remainder being:

$$\pm a_0\mp a_1+\ .\ .\ .\ .\ .\ .\ +a_{n-2}-a_{n-1}+a_n$$

For the third part: If neither $N+1$ nor $N-1$ is divisible by the divisor, d, assume

$$N-r=d\ .\ q.$$

By a method the same as that of the proof of the first part of this rule we arrive at the following algebraic remainder:

$$f(r)=a_0r^n+a_1r^{n-1}+\ .\ .\ .\ .\ .\ .\ +a_{n-1}r+a_n.$$

Since $r = N - dq$, unless $r = \pm 1$ we cannot apply the sum or the difference rule to that divisior in the given number-base system.

This general rule will enable us to determine quickly possibilities of applying sum or difference rules to numbers without going through the labor of finding successive algebraic remainders.

If any number is represented by expression (1), we can rearrange this expression so that it will represent the same number with a base N^2. Thus (1) can be written:

$$(2) \qquad f(N) = F(N^2) = (a_n + Na_{n-1})N^0 + (a_{n-2} + Na_{n-3})N^2 + \ldots\ldots$$

If the coefficients of any number are paired as in (2), we can find divisibility rules for numbers which are factors of $N^2 - 1$ and $N^2 + 1$. This process may be extended by rearranging the number so that it becomes $F(N^3)$, etc. . . , in which the tests applied require that the coefficients be grouped in unit numbers according to the exponent of N in the function.

If this method is applied to the 10-base number system, many otherwise impossible rules are formed. If we have F(100) the conditions will then be:

$$N = 100, \qquad N - 1 = 99, \qquad N + 1 = 101.$$

The factors of 99 are 3, 9, 11, 33, 99. The rule, then, for each of these numbers is the sum rule applied to the digits taken two at a time. The number 101 is prime. The rule to be applied to it is the difference rule to the digits taken two at a time.

In a similar manner, if we have F(1000) we find:

Factors of 999: 3, 9, 27, 37, 111, 333, 999.

Rule: Sum rule applied to digits taken three at a time.

Factors of 1001: 7, 11, 13, 77, 91, 143, 1001.

Rule: Difference rule applied to digits taken three at a time.

This reveals rules for 7 and 13, numbers which could not be tested before without a sequence. Using the same method a rule can be worked out for the divisor 5 in the 7-base system. for which we previously used a sequence.

In conclusion, the remainder theorem can be used to develop rules for divisibility of the sum and difference type, and for some numbers it can be used to develop sequence rules. The sequence rules, however, are usually impractical.

Using the Mathematics News Letter

By Marie L. Renaud
Bay St. Louis, Miss.

Each issue of the *News Letter* contains valuable material for both teacher and student alike. It is an excellent medium for the exchange of ideas in teaching. I, for one, would be lost without my file of *News Letters* in which I constantly delve.

Some time ago I told my plane geography class that the trisection of an angle by means of ruler and compasses was impossible. That surprised them and they demanded to know why. I brought to class the next day the November-December 1931 issue of the *News Letter* and we went over together the excellent article "The Angle-Trisection Chimera Once More."

Every year I assign short papers concerning the history of algebra and geometry to my advanced algebra and geometry classes respectively. I put my copies of the *News Letter* on the reference shelf for students' use. Needless to say many of my topics are taken from various articles in the *News Letter*. They have been very helpful to me in planning my work. One such article is "Major Objectives in Geometry" by F. A. Rickey in the January 1931 issue. Other articles have impressed upon me the fact that no high school mathematics student should be ignorant of the famous men in mathematics and the importance of each—and that no opportunity should be missed in making mathematics an interesting and vital subject. The connection between mathematics and other subjects such as music, art and architecture, should be given and discussed with the students. They will long remember that Einstein is an able violinist as well as scientist and mathematician or that C. L. Dodgson (Lewis Carroll) was a mathematician as well as the author of "Alice in Wonderland." Perhaps such unusual facts will be retained when synthetic division is forgotten!

The references given by Professor R. C. Archibald of Brown University in his article "Humanizing Mathematics" (Nov. 1932) offers a wealth of material in relating mathematics to other subjects in addition to citations of papers on certain values of mathematics.

From time to time I give a fifteen-minute talk to my advanced algebra and geometry classes on some mathematician or some story related to mathematics such as a discussion of Newton's Law of Gravi-

tation or Kepler's laws. Or I might use one of the examples given by Gaston Bruton in his article "Certain Problems" (Dec. 1932) for recreation or "to revive a sleepy class!" These little lectures vary the class routine and serve to stimulate the teacher to find some interesting facts and problems. The classes look forward to there lectures and often discuss the problem given them for days afterwards.

I have mentioned just a few of the articles from which I have gathered ideas and inspiration. I am a "cover to cover" reader of the *News Letter*. Those I have mentioned above serve as examples of the real, practical use that I receive from the *News Letter*.

Secretary Cairns Congratulates S. I. M. A. Founders

December 8, 1933.

My dear Dr. Maizlish:

I have been very much interested in learning, especially through the *Mathematics News Letter*, of the organization of your Southern Intercollegiate Mathematics Association. You and your associates are very much to be congratulated upon the vision which you have of the possibilities of inspiring the teachers and students of your colleges with a new notion of high scholarship. It seems to me that you will have a high degree of success because you have organized the procedure of the association along very novel and evidently very effective lines. I wish to congratulate you on the outlook and to express my hope for the very best of success in this new venture.

Yours very truly,

W. D. CAIRNS, *Secretary-Treasurer,*
Mathematical Association of America.

Book Review Department

Edited by
P. K. Smith

Differential Equations, by L. R. Ford of Rice Institute, is one of several of its class to appear after quite a long silence on the subject in this country. It has 263 6x9 pages arranged in eleven chapters.

Chapter I gives definitions of a number of terms needed in this field, such as differential equation, solution of a differential equation, separation of variables, arbitrary constants, families of curves, field, etc., and illustrates these with examples. The discussion seems to be intended to orient the student in the use of these terms. The reviewer would like to have seen a little more attention given here to the actual origin of such differential equations as the student will meet in his studies in science. This would arouse interest and give purpose to the beginner. However, Professor Ford has followed the prevailing custom in this matter.

Chapter II covers concisely the essentials of the traditional formal methods of integrating equations of the first order. This arrangement is well taken. For it seems adequate for the purpose and reduces the now unwarranted traditional emphasis on this kind of work to more reasonable proportions. The treatment of the linear fractional equation is to be noticed in particular.

Chapter III outlines quite a body of material on linear differential equations of the second order. Linear dependence of solutions of the reduced equation, distribution of "roots" of solutions are discussed. The method of undetermined coefficients, and the method of variation of parameters are given for finding the particular integral of the complete equation, reduction of order, exact equations, and integrating factors all receive attention. The last topic of the chapter, adjoint equations, as treated will hardly be of service to the student, and might have been omitted except for the practical negative information it conveys. In this chapter where the author used the term "roots of solutions," the reviewer would prefer to use "zeros of solutions." For, he believes that the traditional distinction between root of an equation and zero of a function is well worth preserving, and especially so since there is no gain in brevity of one over the other.

Chapter IV outlines the existence theorem of Picard, and the Cauchy-Lipschitz theorem for the equation dy/dx equals f(x,y), and shows how the solution is a continuous function of the initial values.

Chapter V treats systems of differential equations, mostly of the first order, and total differential equations. The conditions of integrability are discussed.

Chapters VI and VII are concerned with such material as is treated by some under the head of "Calculus of observations." These chapters should be useful to students in fields where such methods are employed. Here are given methods of calculating approximate numerical solutions of equations of certain types.

Chapter VIII treats the general linear equation and the independence of systems of solutions of the reduced equation by use of the Wronskian and the Grammian. The particular integral of the general equation with constant coefficients is found by the method of undetermined coefficients, by the method of variation of parameters, and by the symbolic method. The chapter closes with an article on each of: reduction of order, Euler's equation, and systems of linear equations.

Chapter IX discusses solutions in series, and applies the method to some classic equations, such as, the equation of the hypergeometric series, Legendre's equation, and Bessels equation.

Chapter X takes up partial differential equations of the first order, discussing types of solutions, and geometric interpretation. Considerable attention is given to the latter, and several diagrams, new in books of this class, are inserted to aid the imagination.

Chapter XI takes up a few important cases of partial differential equations of the second order, and illustrates their use by some examples in vibrations, and in heat conduction. The reviewer agrees with the author's statement in his preface to the effect that partial differential equations is a vexatious problem in a first course. There may be reasonable doubt if it is less so in its more advanced stages. In view of the present state of the subject Professor Ford has made an excellent presentation.

On the whole the book is a reliable and sufficiently comprehensive introduction to the field of elementary differential equations. The author has given appropriate attention to the logical side of the subject and his book should appeal to students of mathematics. The reviewer

believes that more problem material of the kind that appeals to engineers and science students would improve the book from that stand point. The book may be regarded as a contribution to the pedagogy of elementary differential equations.

The pages are attractive and the general mechanical make-up of the book is excellent.

—*W. Paul Webber.*

Problem Department

Edited by
T. A. Bickerstaff

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

Problems and solutions will be credited to their authors.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

PROBLEMS FOR SOLUTION

No. 49. Proposed by H. T. R. Aude, Colgate University.

Find by construction the circle which fulfills the three conditions: Its center lies on a given line, it is orthogonal to a given circle, and it passes through a given point.

No. 50. Proposed by H. T. R. Aude, Colgate University.

Two circles are orthogonal to each other. One circle cuts the line through the centers in the points A and B. The other circle cuts it in the points C. and D. Show that the points A and B separate the points C and D harmonically.

SOLUTIONS

No. 47. Proposed by E. C. Kennedy, University of Texas.

Prove or disprove:

$$\lim_{N \to \infty} \frac{\sqrt{0}+\sqrt{1}+\sqrt{2}+\sqrt{3}+\,.\,.\,.\,.\,.+\sqrt{n}}{(n+1)\sqrt{n}} = \frac{2}{3}$$

Solved by George A. Baker, Shurtleff College, Alton, Ill.

See Goursat-Hedrick *Mathematical Analysis*, Volume I, page 204, Exercise 2. The general statement is made that the limit of the sum,

$$\sum_{i=0}^{n} \Phi\,(i,n),$$

as n becomes infinite is equal to a certain definite integral whenever $\Phi(i,n)$ is a homogeneous function of degree -1 in i and n.

In this case, the limit of the sum is

$$\int_0^1 \sqrt{y}\,dy = \frac{2}{3}\,y^{3/2}\Big|_0^1 = \frac{2}{3}$$

Also solved by the proposer.

Old Uncle Ned Scattered Butterbean Hulls Before His Doorway

Then he sat back, smoke curling lazily over his corncob pipe, and waited for lightfooted Lady Luck to come tripping, blessings in her hands. "Hit's jus' dat way. Dose butterbean hulls'll call her, so she'll be sho' to come," opined Uncle Ned.

But those days of old have gone, and with them many of the patterns of thinking and living set like so many delicate cameo outlines in the tapestry of past years.

You know that you don't have to sit back and await the whims of a half-seen spirit world. You know that you have the right to work your way to the threshold of Lady Luck and that for your tributes of intelligence and effort honestly offered at the altar of proper ambition, she will come tripping, blessings in her hands. The Louisiana State University will be glad to assist you. For catalog and specific information, write *The Registrar*.

Louisiana State University

Baton Rouge